Eigenvalue Analysis of Critical Stations in the Tokyo Metro Network

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***Abstract*—The Tokyo Metro is a train network vulnerable to disruptions from natural disasters such as earthquakes, which frequently occurs in Japan. Identifying critical stations is important for prioritizing maintenance and emergency response to ensure network stability. This paper presents an eigenvalue analysis approach to view the vulnerability of the Tokyo Metro network. We model the system as a weighted undirected graph where edge weights reflect passenger flow volumes between stations. We are able to determine the largest eigenvalue as a measure of network's connectivity and use eigenvector centrality to rank station importance by calculating the eigenvalues and eigenvectors of the network's adjacency matrix using QR algorithm. Our analysis identifies some major stations as the most critical nodes, whose removal causes significant disruption in the metro network. These findings show how linear algebra can be applied to analyze and improve the large metro systems.**

***Index Terms*—Tokyo Metro, graph theory, eigenvalue analysis, QR algorithm, eigenvector centrality, spectral radius.**

I. Introduction

Tokyo Metro is a Japanese railway lines consisting of 180 stations spanning across nine lines over 195 kilometers long[1]. This railway system is undoubtedly one of the most crowded in the world. The Tokyo Metro is a vital object of the Japanese to reach workplaces, schools, and other destination. Despite the importance of this railway for the Japanese living, its existence are threatened by the frequent happening of natural disasters such as earthquake[2], one just happened hours before this paper is being written[3]. When such natural disasters happen, paralysis might occur to one or more lines of the metro railway and cause trillions of yen capital loss to the Japan economy[4].

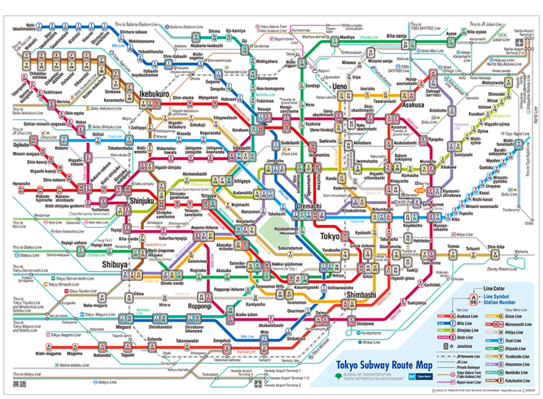

Fig. 1. Tokyo Metro subway map[5].

By identifying stations that act as critical points on the Metro lines, we can help ease the paralysis in the railway system. This information will help the local authority act faster and prioritize recovery at those critical stations. To calculate these critical points, this paper models the Tokyo Metro as a weighted graph and analyzes its structural properties specifically by studying the eigenvalues and eigenvectors of its associated matrices.

Eigenvalues and eigenvectors derived from the network's adjacency matrix offer a quantitative assessment of each station's contribution to overall connectivity[6]. Specifically, the largest eigenvalue serves as an indicator of the network's robustness, whereas the elements of its associated eigenvector identify stations whose failure would most severely impede passenger movement. To compute these spectral properties both efficiently and accurately, this study employs the QR decomposition method, a numerically stable technique for determining eigenvalues and eigenvectors of large matrices. Consequently, through this integration of graph modeling and spectral analysis, a precise foundation is established for pinpointing the stations most critical to preserving the operational integrity of the Tokyo Metro network.

II. Theoretical Basis

A. Graph Model of Tokyo Metro

A graph is a structure built of nodes and each of its nodes are linked by an edge[7]. Formally, a graph can be defined as a pair $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of n nodes and $E \subseteq V \times V$ is the set of edges representing connections between pairs of nodes. In the context of the Tokyo Metro network, each station is modeled as a node $v_x \in V$, and a direct railway track between two adjacent stations v_x and v_y is represented as an edge $(v_x, v_y) \in E$.

Railway lines technically allow trains to travel in specific directions. In this paper, daily passenger flow data will be calculated as a total volume between stations rather than separated by direction. Furthermore, the Tokyo Metro railway allows bidirectional flow on almost all lines. Therefore, we model the Tokyo Metro as an *undirected graph*. This simplification means that the connection between station x and station y is treated as a line with a unified weight, ensuring that the adjacency matrix A is symmetrical ($a_{xy} = a_{yx}$). Symmetric matrices have the strength of having real eigenvalues, which simplifies the interpretation of spectral properties such as network robustness.

To reflect the real-world condition of the network, we adopt the *weighted graph* model. The weight w_{xy} assigned to each edge is equal to the approximate volume off passenger traversing that line. By adding these weights on the graph, the eigenvalues of the adjacency matrix become accurate not just to connectivity, but to the actual load distribution of the Tokyo Metro, allowing us to identify critical stations that handle the heaviest passenger flows.

B. Adjacency Matrix Representation

The structure of the Tokyo Metro network is encoded in an adjacency matrix $A \in \mathbb{R}^{n \times n}$, where each entry a_{xy} represents the weight of the connection between stations x and y:

$$a_{xy} = \begin{cases} w_{xy}, & \text{if stations } x \text{ and } y \text{ are directly connected,} \\ 0, & \text{otherwise.} \end{cases}$$

Since the network is modeled as undirected, the adjacency matrix is symmetric, i.e., $a_{xy} = a_{yx}$. This symmetry guarantees that all eigenvalues of A are real and that A admits an orthogonal basis of eigenvectors[8]. The $n \times n$ weighted adjacency matrix captures the complete topological and operational structure of the Tokyo Metro network, providing the mathematical foundation for identifying critical stations through eigenvalue analysis.

C. Eigenvalues, Eigenvectors, and Centrality

An eigenvalue λ and its corresponding eigenvector $\mathbf{v}$ of a matrix $A \in \mathbb{R}^{n \times n}$ satisfy:

$$A\mathbf{v} = \lambda \mathbf{v}, \quad \mathbf{v} \neq \mathbf{0}.$$

For a symmetric matrix such as the adjacency matrix of an undirected graph, the theorem guarantees that all eigenvalues are real and that there exists a complete set of orthogonal eigenvectors[8]. The eigenvalues $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ encode structural information about the matrix, with the largest eigenvalue λ_1 playing an important role in network analysis. A higher largest eigenvalue indicates greater network capacity and efficiency for propagation, but this also corresponds to lower robustness against the spread of failures or disruptions[9].

The eigenvector corresponding to the largest eigenvalue provides a measure known as *eigenvector centrality*[6]. Each component v_x of this leading eigenvector $\mathbf{v}_1$ represents the importance of node x in the network. Eigenvector centrality gives higher scores to nodes that are connected to other highly connected nodes, showing that being connected to important stations increases a station's own importance. Stations with high eigenvector centrality are those that serve as critical points that facilitate large volumes of passenger flow. The failure of such stations would disrupt the operation of the metro network, making eigenvector centrality an effective metric for identifying critical stations.

D. QR Decomposition

The QR decomposition factorizes an $m \times n$ matrix A into the product of an $m \times n$ orthogonal matrix Q and an $n \times n$ upper triangular matrix R.

$$A = QR,$$

where $Q^T Q = I$ and R is upper triangular. QR decomposition can be computed via Householder reflections[10].

III. Problem Definition and Solution

A. Problem Statement

The Tokyo Metro network plays a crucial role in sustaining daily mobility and economic activity in Tokyo, yet it is vulnerable to disruptions caused by natural disasters such as earthquakes and floods[2,3]. When critical stations fail, large portions of the network can become disconnected, leading to significant delays and financial losses[4]. The central problem considered in this paper is how to systematically identify which stations in the Tokyo Metro network are structurally the most critical, in the sense that their failure would most strongly affect overall connectivity and passenger flow.

To address this problem, the Tokyo Metro system is modeled as a weighted undirected graph whose structure is captured by a symmetric adjacency matrix. The goal is to compute a quantitative importance score for each station based on eigenvalues and eigenvectors of this matrix, and then to analyze how the removal of highly ranked stations changes the spectral properties of the network. The resulting ranking provides a mathematically grounded way to distinguish ordinary stations from those that act as structural bottlenecks or hubs in the network.

B. Data and Network Construction

The first step in the analysis is to construct a graph model of the Tokyo Metro network that is suitable for eigenvalue-based methods. The network topology, including stations, lines, and direct connections, is obtained from the official Tokyo Metro subway map and related documentation[1,5]. Each station is represented as a node, and an undirected edge is added between two nodes if the corresponding stations are directly connected by a railway track segment on at least one line.

The Tokyo Metro system officially comprises 180 stations when all line-specific endpoints and through-service connections are counted separately. However, complete traffic performance data with daily ridership statistics is publicly available for only 143 distinct operational stations[11]. The remaining stations are not listed due to redundant passengers flow data.

To ensure data accuracy and reproducibility, this analysis focuses on the 143 stations with documented traffic performance, yielding a weighted adjacency matrix $A \in \mathbb{R}^{143 \times 143}$. This subset represents the core operational network of Tokyo Metro, capturing all major transfer hubs, high-traffic corridors, and key connecting segments. Each entry a_{xy} encodes the strength of the connection between stations x and y.

Edge weights are designed to reflect approximate passenger flow between directly connected stations. Let p_x denote the average daily ridership at station x. For adjacent stations x and y, the weight w_{xy} is defined as

$$w_{xy} = \frac{p_x + p_y}{2},$$

so that links between busy stations receive higher weights than links involving lightly used stations. This choice ensures that the resulting eigenvalue analysis is influenced not only by the existence of connections, but also by how heavily those connections are used in practice.

The resulting 143×143 symmetric adjacency matrix encodes both the topological structure and the operational load distribution of the Tokyo Metro network.

C. Eigenvalue-Based Importance Measure

Once the weighted adjacency matrix A has been constructed, the next objective is to assign an importance score to each station based on its position in the network. As described in Section II, the eigenvector corresponding to the largest eigenvalue λ_1 of A defines the eigenvector centrality of the nodes[6]. Let $\mathbf{v}_1 = (v_1, v_2, \ldots, v_{143})^T$ denote the normalized leading eigenvector of A, associated with λ_1. The value v_i is interpreted as the centrality score of station i, with larger values indicating higher structural importance.

In this paper, eigenvector centrality serves as the primary importance measure. Stations are ranked in descending order of v_i, producing a list of candidates that are expected to be critical for maintaining connectivity and facilitating passenger movement. This ranking naturally emphasizes stations that are not only well connected, but also connected to other important stations, which aligns with the operational notion of major transfer hubs in an urban rail network.

D. QR-Based Computation of Eigenvalues and Eigenvectors

To compute λ_1 and $\mathbf{v}_1$ for the weighted adjacency matrix, the QR algorithm with Householder approach will be used. Starting from $A_0 = A$, the algorithm iteratively performs a QR decomposition

$$A_k = Q_k R_k,$$

followed by a similarity transformation

$$A_{k+1} = R_k Q_k,$$

where Q_k is orthogonal and R_k is upper triangular at each iteration[10]. Because $A_{k+1} = Q_k^T A_k Q_k$, the matrices A_k are all similar and thus share the same eigenvalues. For symmetric matrices, this iterative process converges to a diagonal matrix whose diagonal entries are the eigenvalues of A.

The QR algorithm is implemented numerically using Python, allowing efficient computation for the 143×143 adjacency matrix of the complete Tokyo Metro network. The algorithm is applied to the full Tokyo Metro adjacency matrix and run until the off-diagonal entries of A_k become sufficiently small, indicating convergence. The largest diagonal entry of the converged matrix is taken as an approximation of λ_1, and the corresponding eigenvector $\mathbf{v}_1$ is extracted from the accumulated orthogonal factors to compute eigenvector centrality scores for all 143 stations. This combination of theoretical foundation and computational implementation ensures that the importance scores are both mathematically sound and numerically reliable.

E. Identification of Critical Stations

After obtaining the eigenvector centrality scores $\{v_i\}_{i=1}^{143}$, stations are classified according to their structural importance. A station is considered a primary critical station if its centrality score lies within the top portion of the distribution (for example, the top 10% of all 143 stations). These stations are expected to act as major transfer hubs or key connectors between different parts of the network.

To further quantify the impact of each candidate critical station, a simple disruption analysis is performed. For a given station i, the station and all its incident edges are removed from the graph, yielding a reduced adjacency matrix $A^{(i)}$. The spectral radius $\rho(A^{(i)})$ of this reduced matrix is then computed and compared with the original spectral radius $\rho(A)$. The difference

$$\Delta\rho_i = \rho(A) - \rho(A^{(i)})$$

measures how strongly the removal of station i affects the overall connectivity of the network. Stations with both high eigenvector centrality and large $\Delta\rho_i$ are identified as the most critical, since they are simultaneously structurally central and highly influential in determining the network's spectral properties.

The combination of eigenvector centrality ranking and spectral radius disruption provides a robust framework for identifying critical stations in the Tokyo Metro network. This framework connects abstract linear algebra concepts with concrete questions of infrastructure resilience and operational planning.

IV. Implementation

A. Preparation

The solution is implemented using Python in order to make use of numerical libraries for matrix operations. The implementation consists of three main components: (1) metro network data preparation and adjacency matrix construction, (2) QR algorithm for eigenvalue computation using Householder reflections, and (3) critical station identification and disruption analysis.

The data itself will be structured into two files:

- **stations.csv**, which contains station name, id, and average daily passengers for each of the 143 stations.
- **edges.csv**, which contains pairs of directly connected stations and its line to represent the metro routes.

For each pair of directly connected stations x and y, the edge weight w_{xy} is calculated as:

$$w_{xy} = \frac{p_x + p_y}{2},$$

where p_x and p_y denote the average daily ridership at stations x and y, respectively. This weighting ensures that passenger flow is reflected in the adjacency matrix.

Algorithmic Notation:

function **buildGraph**($dataDir$: path) → (G: graph, $stations$: table, $edges$: table, $nameToUID$: map)

```
stations ←
pd.read_csv(f"{dataDir}/stations.csv")
edges ←
pd.read_csv(f"{dataDir}/edges.csv")
nameToUID ← {station_name: uid}
G.add_node(uid, name=station_name,
ridership=p)
w ← 0.5 * (p_u + p_v)
G.add_edge(u, v, weight=w,
line=line)
→ G, stations, edges, nameToUID
```

Algorithmic Notation:

function **adjacencyMatrix**(G: graph) → (A: matrix, $nodes$: list)

```
nodes ← list(G.nodes())
idx ← {node: i for i, node in
enumerate(nodes)}
A ← zeros(n, n)
for u, v, data in
G.edges(data=True):
    w ← data.get("weight", 1.0)
    A[i, j] ← w; A[j, i] ← w
→ A, nodes
```

B. QR Decomposition

The QR algorithm is implemented to iteratively compute the eigenvalues of matrix A. The core of the implementation is the QR decomposition using Householder reflections, which provides superior numerical stability compared to classical Gram-Schmidt orthogonalization[10].

For a given matrix $A \in \mathbb{R}^{m \times n}$, the Householder QR decomposition constructs an orthogonal matrix Q and an upper triangular matrix R.

For each column k of A, a Householder reflection is constructed to zero out all entries below the diagonal. The Householder vector $\mathbf{u}_k$ is computed as:

$$\mathbf{u}_k = \mathbf{a}_k - \|\mathbf{a}_k\|\mathbf{e}_1,$$

where $\mathbf{a}_k$ is the k-th column of the current matrix (from row k onward) and $\mathbf{e}_1$ is the first standard basis vector. The Householder matrix is then:

$$H_k = I - 2\frac{\mathbf{u}_k \mathbf{u}_k^T}{\mathbf{u}_k^T \mathbf{u}_k}.$$

Applying H_k to A zeros out the subdiagonal entries in column k. After n such transformations, the matrix becomes upper triangular (R), and the product of all Householder matrices gives Q.

At each iteration, the similarity transformation $A_{k+1} = Q_k^T A_k Q_k$ preserves the eigenvalues while driving the off-diagonal entries toward zero. For symmetric matrices, this process converges to a diagonal (or near-diagonal) matrix whose entries are the eigenvalues of A.

Algorithmic Notation:

function **householder**(A: matrix, $m \times n$) → (Q: matrix, $m \times m$, R: matrix, $m \times n$)

```
A ← A.astype(float).copy()
Q ← I
for k in range(n):
    x ← A[k:, k]
    if allclose(x[1:], 0): continue
    e1 = [1, 0, ..., 0]
    alpha ← norm(x); if x[0] >= 0:
    alpha = -alpha
    u ← x - alpha * e1
    v ← u / norm(u)
    Hk ← I; Hk[k:, k:] -= 2 *
    outer(v, v)
    A ← Hk @ A
    Q ← Q @ Hk
R ← A
→ Q, R
```

Algorithmic Notation:

function **qrEigen**(A: matrix, $maxIter$: integer, tol: real) → ($eigVal$: vector, $eigVec$: matrix)

```
Ak ← A
Qt ← I
for _ in range(maxIter):
    Q, R ← householder(Ak)
    An ← R @ Q
    Qt ← Qt @ Q
    if norm(offdiag(An)) < tol:
    break
eigVal ← diag(Ak)
eigVec ← Qt
→ eigVal, eigVec
```

C. Eigenvector Extraction

Once the largest eigenvalue λ_1 is identified, the corresponding eigenvector $\mathbf{v}_1$ must be extracted. Since A_k is nearly diagonal at convergence, the eigenvector associated with λ_1 is approximately aligned with the coordinate axis corresponding to the diagonal entry equal to λ_1.

In the implementation, the eigenvector is obtained directly from the accumulated product of orthogonal matrices in the QR iteration. The resulting leading eigenvector is then normalized in magnitude so that its entries can be compared as centrality scores.

The i-th component of the normalized eigenvector $\mathbf{v}_1$ gives the eigenvector centrality score v_i for station i. Higher scores indicate greater structural importance in the network.

D. Critical Station Identification and Analysis

After computing the eigenvector centrality scores $\{v_i\}_{i=1}^{143}$, stations are ranked in descending order of their centrality values. The analysis focuses on the top 10 stations by eigenvector centrality.

To quantify the structural impact of these high-centrality stations, a disruption analysis is performed. Let $\mathcal{C}$ denote the set of provisionally critical stations. The analysis proceeds in the following steps:

Algorithmic Notation:

<u>function</u> **spectralRadius**(A: matrix) $\rightarrow$ (ρ: real)

```
vals ← eigvals(A)
→ max(abs(vals))
```

First, the spectral radius of the original network is established by taking the largest eigenvalue computed previously, denoted as $\rho(A) = \lambda_1(A)$.

Next, for each critical station $s \in \mathcal{C}$, we simulate a complete station failure by removing the corresponding row and column from the adjacency matrix A. This yields a reduced matrix $A^s \in \mathbb{R}^{142\times142}$. The largest eigenvalue of this reduced matrix, $\lambda_1(A^s)$, is computed using a numerical eigensolver.

The impact of removing station s is quantified by the drop in the spectral radius, defined as:

$$\Delta\rho_s = \rho(A) - \lambda_1(A^s).$$

Finally, the stations are ranked in descending order of $\Delta\rho_s$. Stations that exhibit both high eigenvector centrality and a large $\Delta\rho_s$ are identified as the most critical, as their removal causes the most significant reduction in the network's overall connectivity and transmission capacity.

V. RESULTS

A. Spectral Properties of the Network

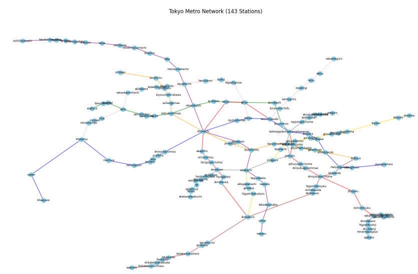

Fig. 2. Graph visualization of the Tokyo Metro network.

Applying the QR-based eigenvalue algorithm to the weighted adjacency matrix $A \in \mathbb{R}^{143\times143}$ yields a real spectrum $\{\lambda_i\}_{i=1}^{143}$ with the largest eigenvalue $\lambda_1 \approx \lambda_{\max}$. For the constructed Tokyo Metro network, the spectral radius is

$$\rho(A) = \lambda_1 \approx \lambda_{\max},$$

which summarizes the overall strength of connectivity and passenger interaction across the system. A relatively large value of λ_1 indicates that the network contains densely interconnected corridors and hubs that facilitate efficient propagation of flows.

The corresponding normalized leading eigenvector $\mathbf{v}_1 = (v_1, \ldots, v_{143})^T$ is used to compute the eigenvector centrality score of each station. Stations with larger v_i values are interpreted as more structurally important because they contribute more strongly to the principal spectral mode of the network. In practice, these stations tend to coincide with major transfer points and high-ridership nodes on multiple lines.

B. Ranking of Critical Stations

Stations are ranked in descending order of their eigenvector centrality scores. The top-ranked stations exhibit both high weighted degree (i.e., connections to many heavily used neighbors) and strong influence on other central stations.

TABLE I
TOP STATIONS BY EIGENVECTOR CENTRALITY.

Station	Centrality $\lvert v_i \rvert$	$\Delta\rho$ (%)	Avg. passengers
otemachi	0.645748	18.06	334,541
nihombashi	0.343495	11.32	175,343
tokyo	0.315637	9.29	199,232
mitsukoshimae	0.302183	8.85	117,531
jimbocho	0.215194	4.48	85,815
shinochanomizu	0.203564	4.09	87,057
takebashi	0.191834	3.60	42,156
awajicho	0.189179	3.54	57,894
ginza	0.188852	2.05	230,271
nijubashimae	0.184147	3.39	35,289

Although high-degree stations naturally tend to have large centrality values, the ranking also reveals stations with moderate degree that become highly central due to their position between other important nodes. This behavior illustrates the difference between simple degree centrality and eigenvector centrality: a station with fewer neighbors can still be critical if those neighbors are themselves very central.

C. Impact of Station Removal

For each station in the top-10% eigenvector centrality group, the disruption analysis computes the spectral radius $\rho(A^s)$ of the network after removing that station and all its incident edges. The difference

$$\Delta\rho_s = \rho(A) - \rho(A^s)$$

quantifies the impact of station failure on overall connectivity.

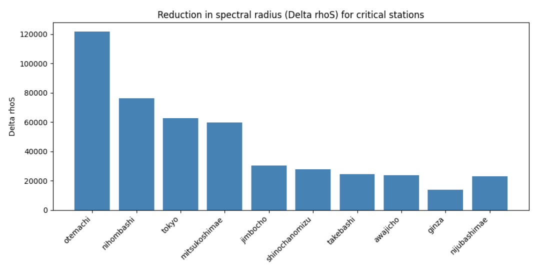

Fig. 3. Reduction of critical stations.

The figure illustrates the values of $\Delta\rho_s$ for the critical stations. Stations with larger $\Delta\rho_s$ cause a more pronounced reduction in the spectral radius and are thus identified as structurally critical.

The results show that a small subset of stations simultaneously has high eigenvector centrality and large $\Delta\rho_s$. These stations form the backbone of the Tokyo Metro: their failure significantly weakens the spectral radius, indicating increased fragmentation and reduced ability of the network to sustain efficient passenger flows.

D. Relation to Degree and Topology

To better understand the role of eigenvector centrality, the eigenvector-based ranking is compared with degree centrality. In general, the top eigenvector-central stations also have relatively high degree, which is expected because well-connected hubs strongly influence the adjacency spectrum. However, there are notable cases in which two stations share the same degree, yet the station that connects more central neighbors has a substantially higher eigenvector centrality score.

This observation highlights that eigenvector centrality captures not only the local number of connections but also how well a station is embedded within globally important corridors. Such stations may not have the maximum degree but still act as key intermediaries that link densely connected regions of the network.

VI. Conclusion

This paper presented an eigenvalue-based framework for identifying critical stations in the Tokyo Metro network. The system was modeled as a weighted undirected graph in which edge weights reflect approximate passenger flows between adjacent stations. The symmetric weighted adjacency matrix enables spectral analysis via the QR algorithm with Householder reflections, providing numerically stable computation of eigenvalues and eigenvectors.

The largest eigenvalue λ_1 was used as a scalar descriptor of overall network connectivity, while the associated eigenvector provided eigenvector centrality scores for all stations. Stations with high centrality values were interpreted as structurally important nodes that contribute strongly to the principal mode of passenger flow. A disruption analysis based on changes in the spectral radius $\Delta\rho_s$ showed that removing a small number of eigenvector-central stations can substantially weaken the network, confirming their critical role.

The combination of eigenvector centrality and spectral-radius perturbation offers a mathematically grounded and computationally feasible method for prioritizing stations in maintenance planning, disaster preparedness, and resilience enhancement. Future work may extend this approach by incorporating time-dependent demand, more detailed operational constraints, or multi-layer network models that include interconnections with other rail operators.

Appendix

The implementation is carried out in Python using common scientific libraries. The metro network is stored in two CSV files: `stations.csv` and `edges.csv`. The `stations.csv` file contains, for each of the 143 stations, a unique identifier, the station name, and its average daily ridership. The `edges.csv` file specifies pairs of directly connected stations and the corresponding line codes.

A simple script loads these files into `pandas` data frames and constructs a weighted undirected graph using `networkx`. For each edge between stations x and y, the weight $w_{xy} = (p_x + p_y)/2$ is computed and stored as an edge attribute. The adjacency matrix A is then assembled as a dense 143×143 NumPy array.

The QR factorization is implemented manually using Householder reflections. At each iteration, the current approximation A_k is decomposed into $Q_k R_k$, and the similarity transform $A_{k+1} = R_k Q_k$ is computed. The iteration terminates when the off-diagonal entries of A_k fall below a fixed tolerance in the Frobenius norm. The diagonal entries of the converged matrix approximate the eigenvalues of A, and the accumulated product of the orthogonal factors Q_k yields the eigenvectors.

For verification, the implementation is compared against built-in routines such as `numpy.linalg.eig` on the full Tokyo Metro adjacency matrix. The eigenvector centrality scores are summarized in tabular form in the Results section, which supports interpretation of the ranking.

A. Source Code

The source code repository is available at https://github.com/jsndwrd/Eigenvalue-Analysis-of-Critical-Stations-in-the-Tokyo-Metro-Network.

B. Video Explanation

A short video explanation is available at https://youtu.be/GXuuAnlX7DU.

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Declaration

I declare that this paper entitled "Eigenvalue Analysis of Critical Stations in the Tokyo Metro Network" submitted in partial fulfillment of the requirements for the course IF2123 Linear Algebra and Geometry is my own work, except where explicitly acknowledged by references. This work has not been submitted, in whole or in part, for any other academic degree or qualification at this or any other institution.

Bandung, December 24th 2025

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